Domain and Range

Objectives

Upon completion of this unit students should be able to:

1. Understand the domain and range of functions

2. Understand the graphical meaning of domain and range

3. Find the domain of rational functions algebraically and graphically

4. Find the domain and range of radical functions algebraically and graphically

5. Find the domain and range of polynomial functions algebraically and graphically.

Consider the following profit function

$$P(x) = -x^2 + 1960x - 50{,}000$$

where x is the number of units produced and sold.
Then the profit from producing and selling 1000 units is given by

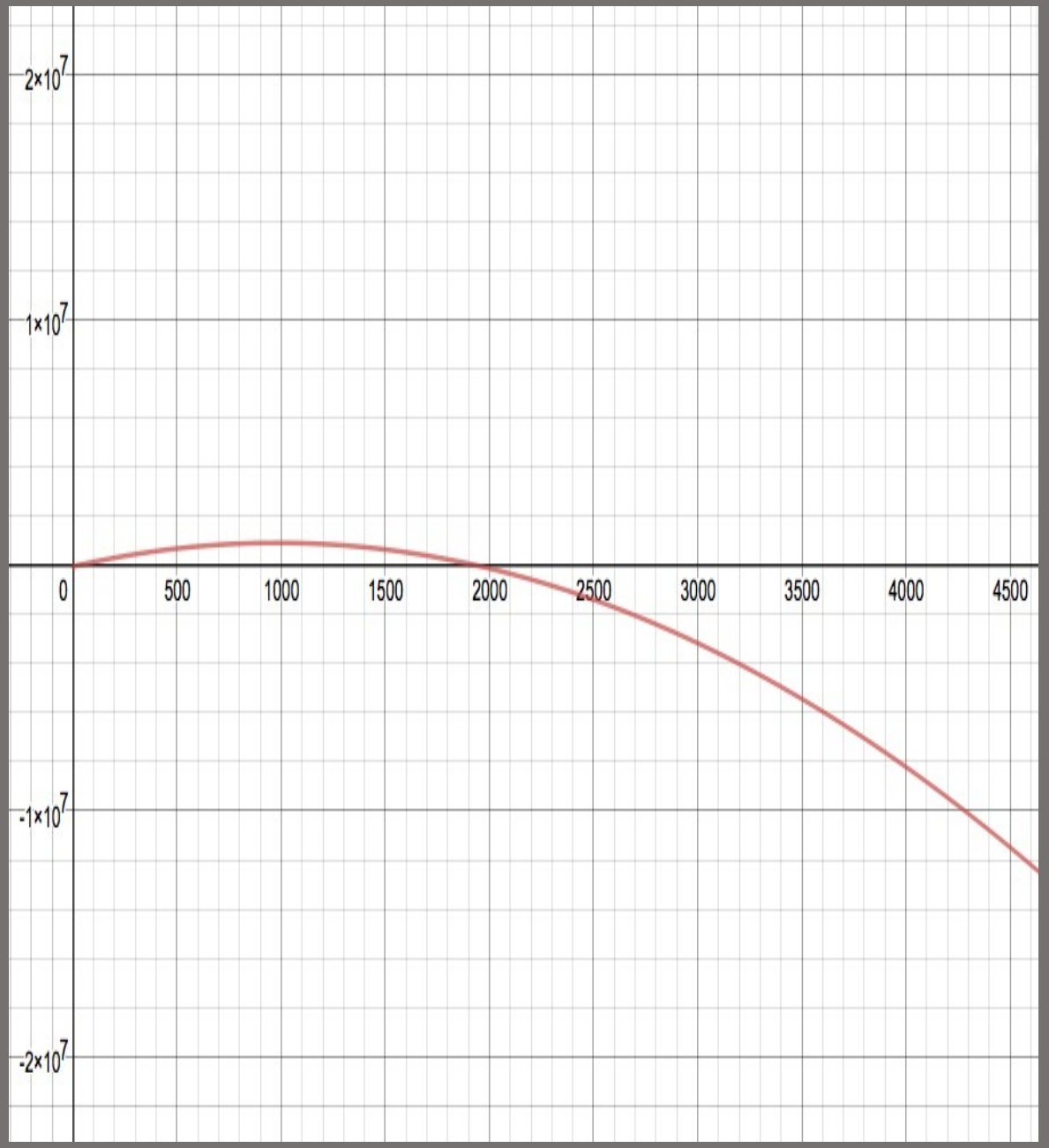
$$P(1000) = -(1000)^2 + 1960(1000) - 50{,}000 = \$910{,}000$$

The profit of 1934 units is

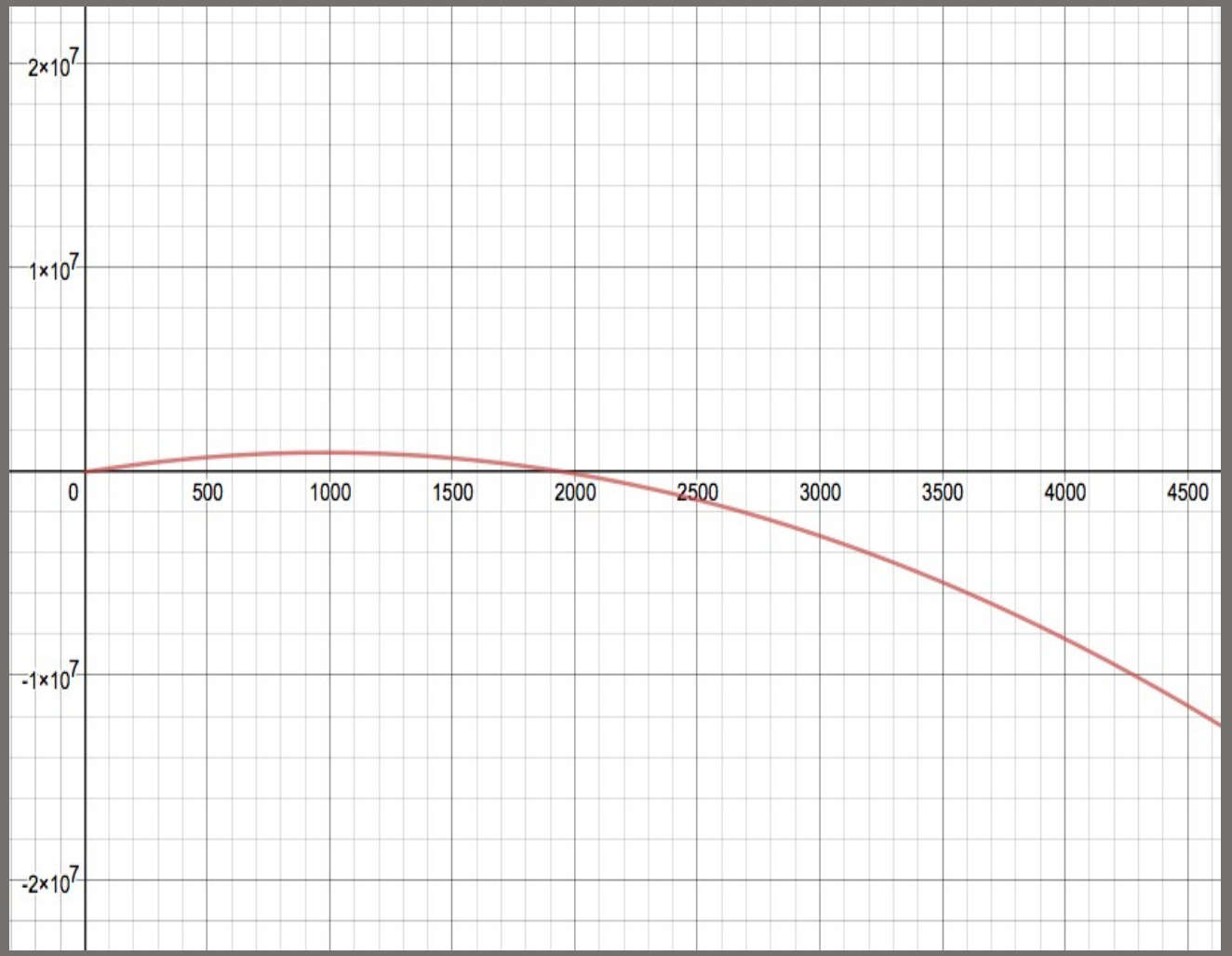
$$P(1934) = -(1934)^2 + 1960(1934) - 50{,}000 = \$284$$

The profit of 1935 units is

$$P(1935) = -(1935)^2 + 1960(1935) - 50{,}000 = \$ - 1{,}625$$



- For this specific profit function does it make sense to plug in a negative number?
- Or a number beyond 1934?
- No since we cannot produce negative units and once we begin producing more than 1934 units it costs us money.
- That is the purpose of a function's domain.
- What are the allowable values that can be plugged into a function?
- We should also be aware of the outputs of such a function. The outputs will tell us how much profit we are making.
- The outputs of a function are the range of the function.



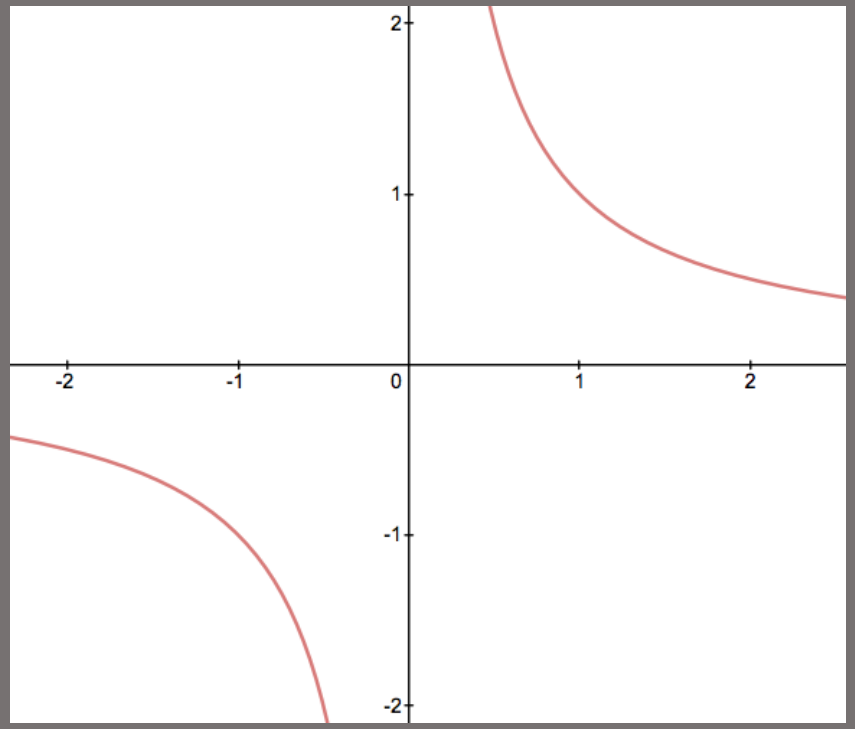
Definitions:

1. The **domain** of a function is the set of all inputs of a function.

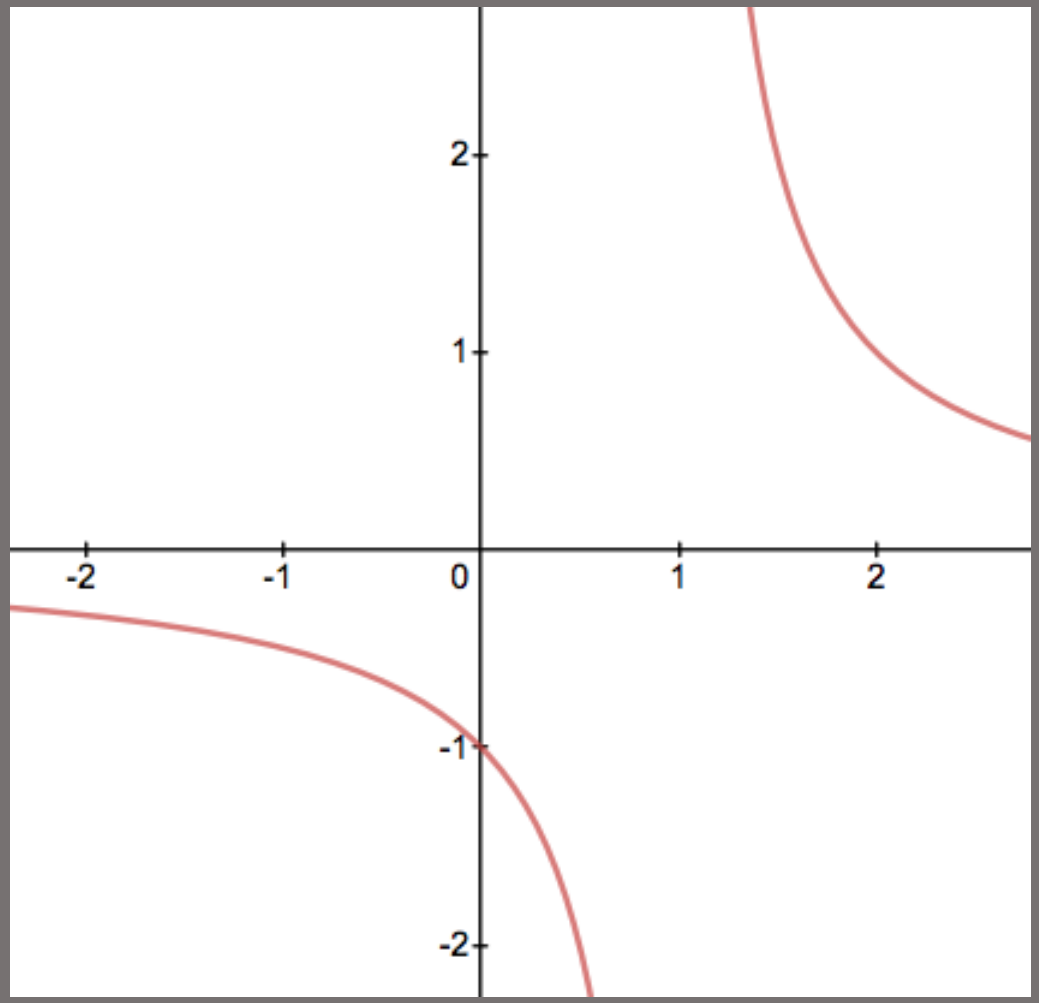
2. The **range** of a function is the set of all output of a function.

Example 1 (Domain of Rational Functions). Find the domain of the following functions.

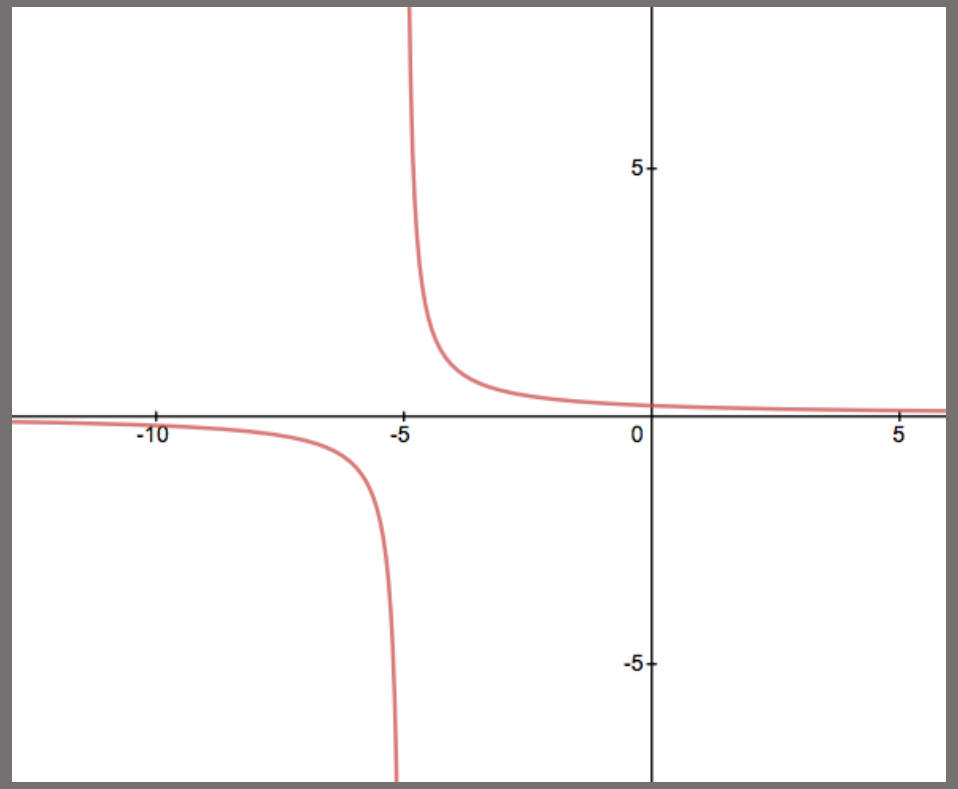
(a) $f(x) = \frac{1}{x}$



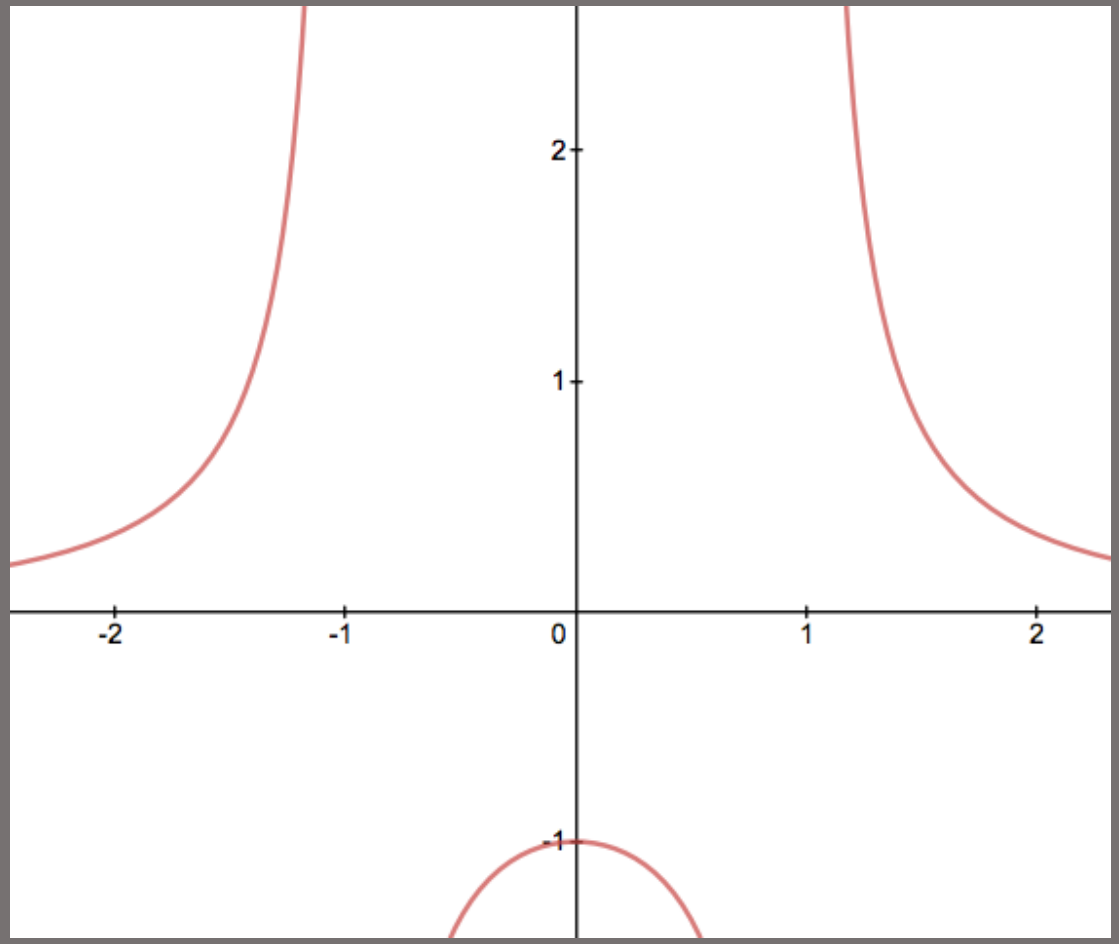
(b) $f(x) = \frac{1}{x-1}$



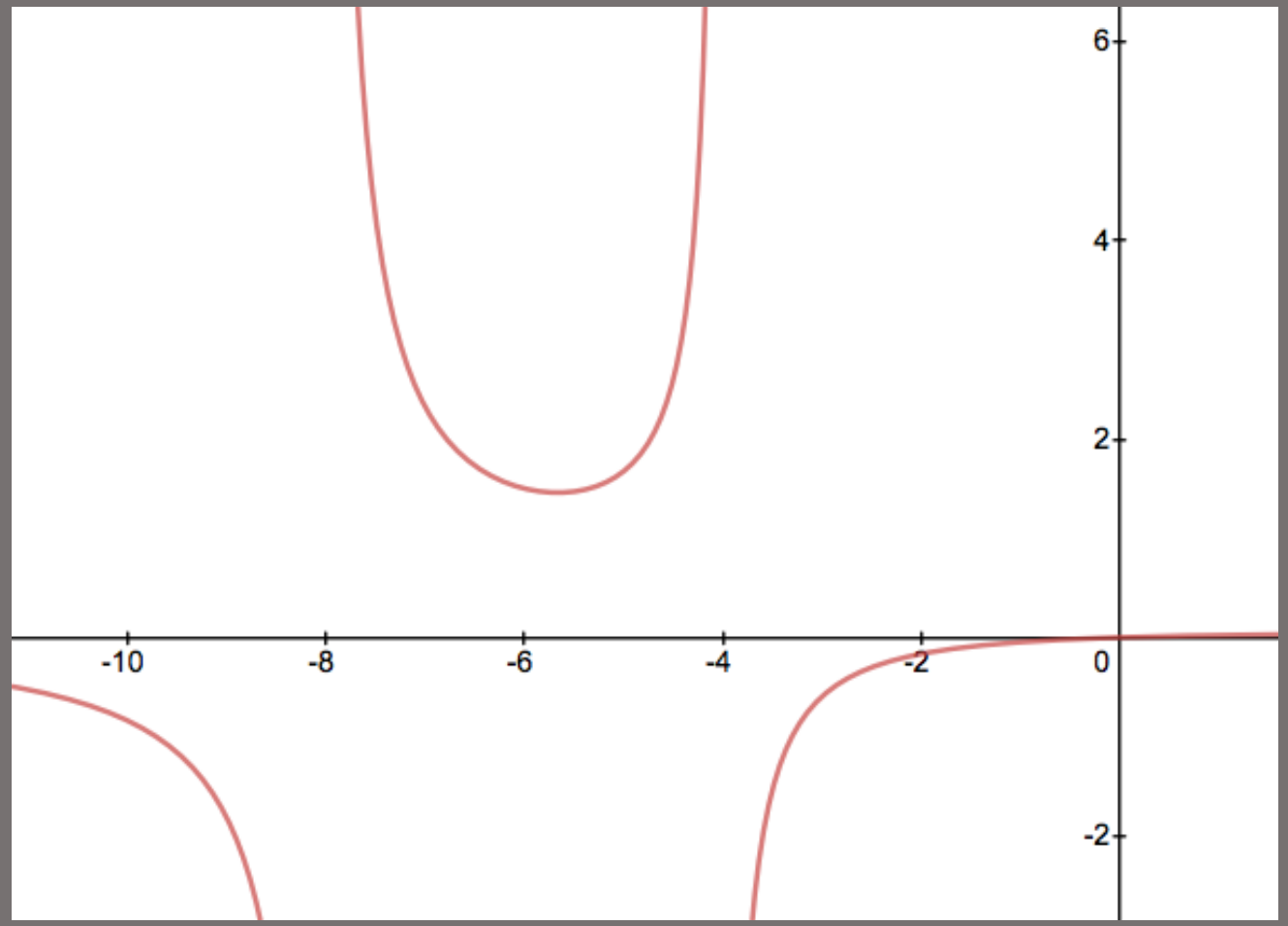
(c) $f(x) = \frac{1}{x+5}$



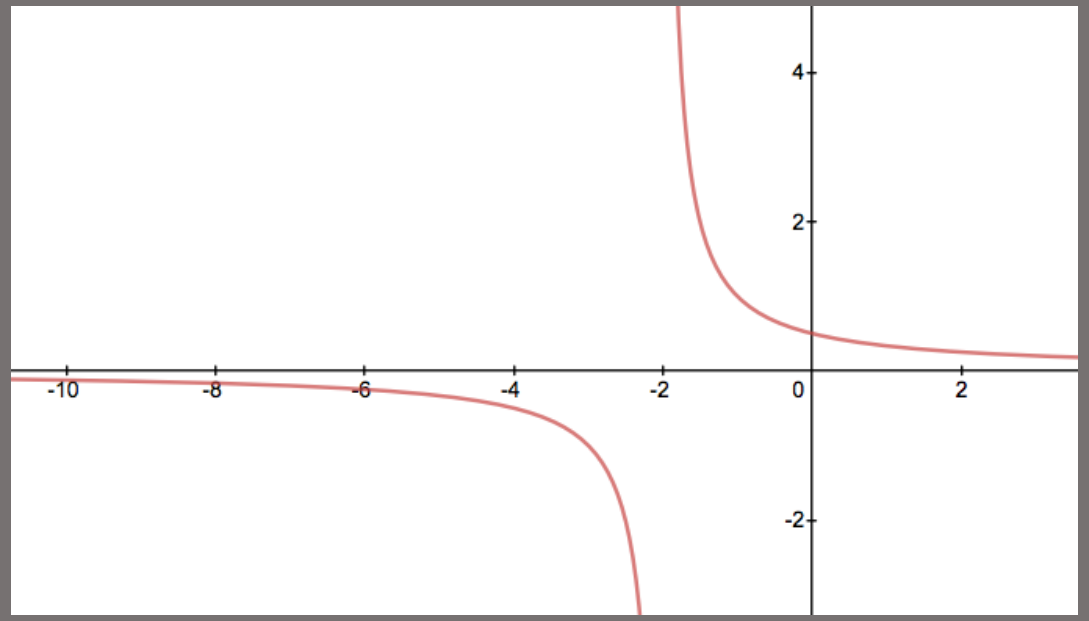
(d) $f(x) = \frac{1}{x^2-1}$



(e) $f(x) = \frac{x}{x^2+12x+32}$

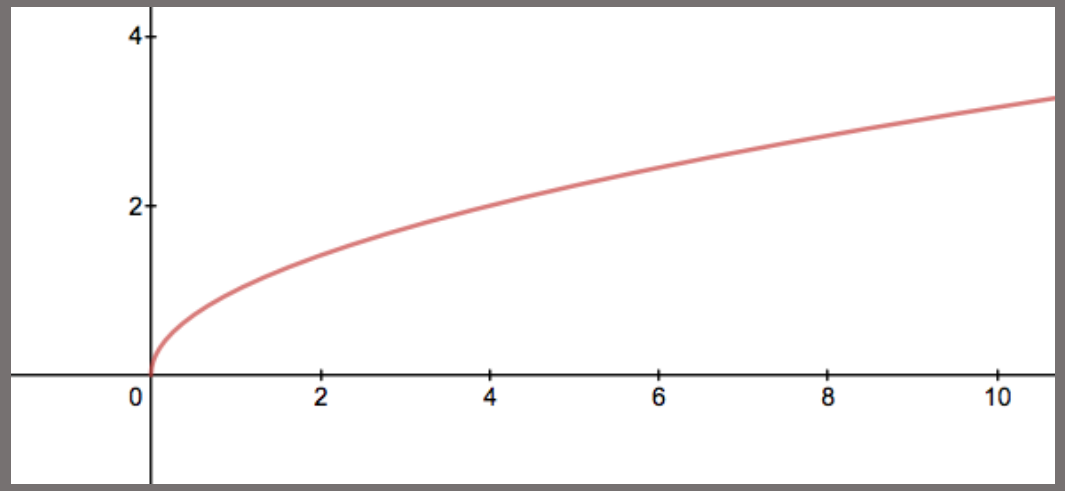


(f) $f(x) = \frac{x-2}{x^2-4}$

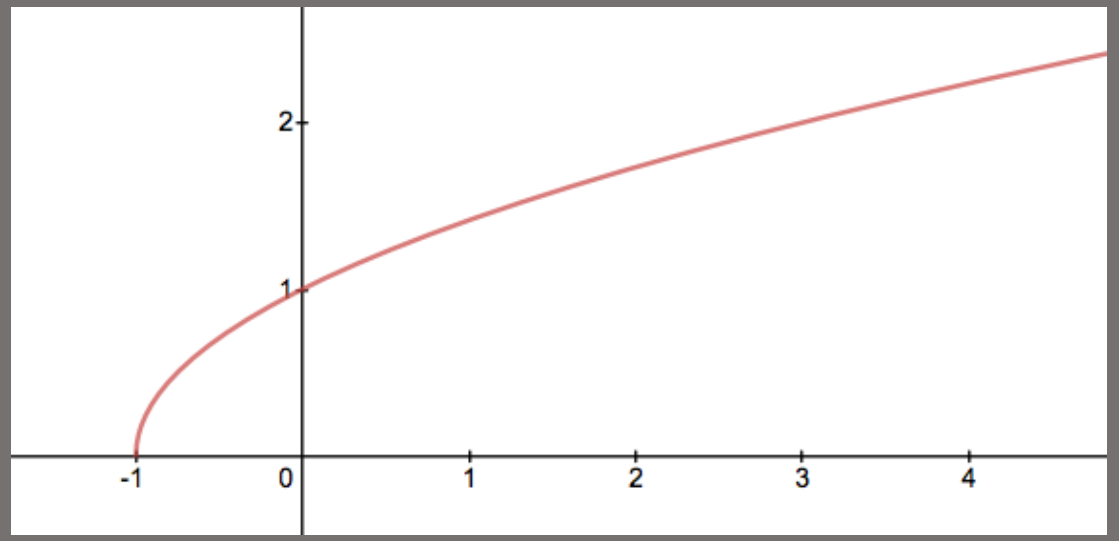


Example 2 (Domain of Radical Functions). Find the domain and range of the following functions.

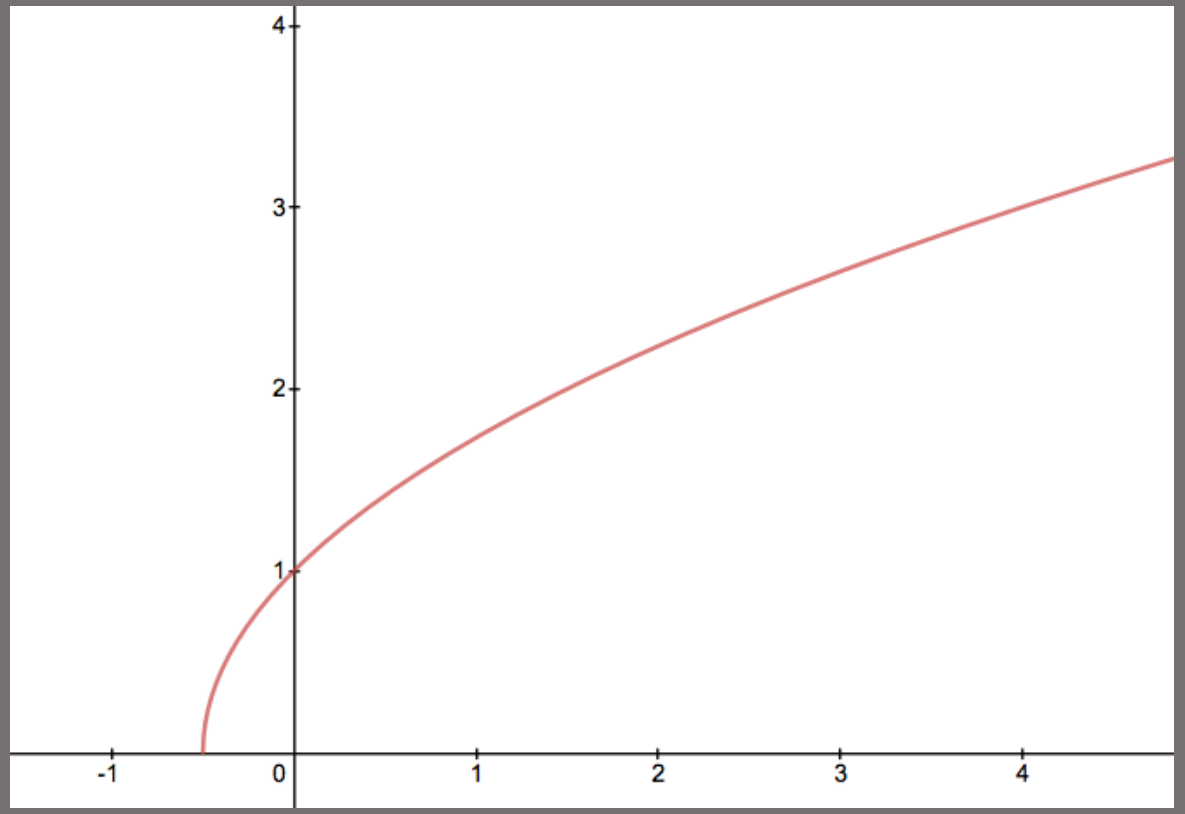
(a) $f(x) = \sqrt{x}$



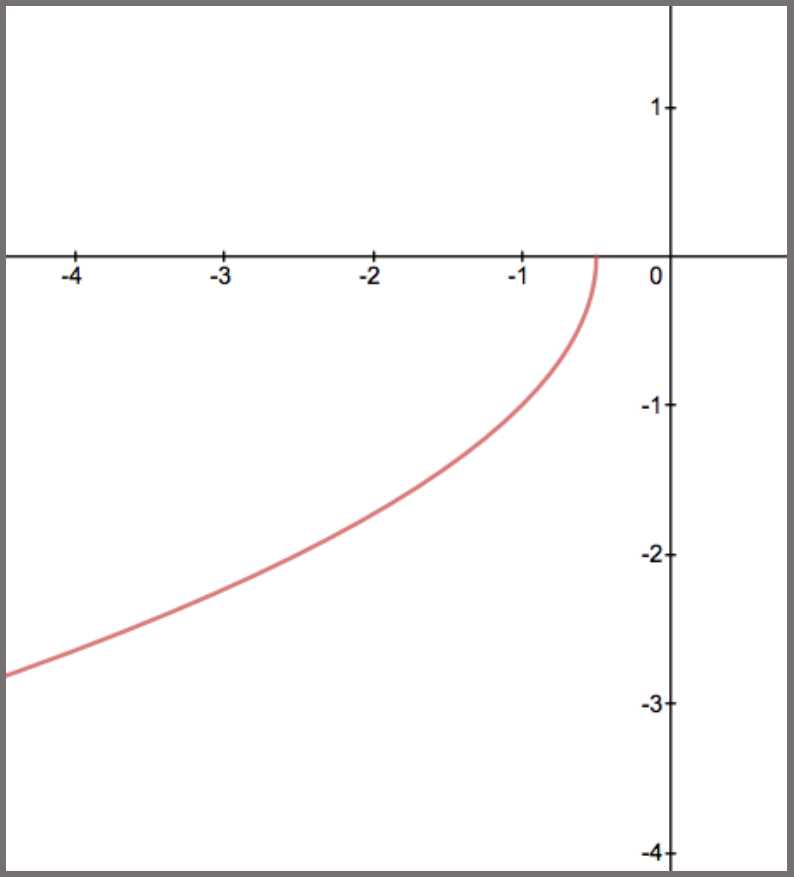
(b) $f(x) = \sqrt{x+1}$



(c) $f(x) = \sqrt{2x + 1}$

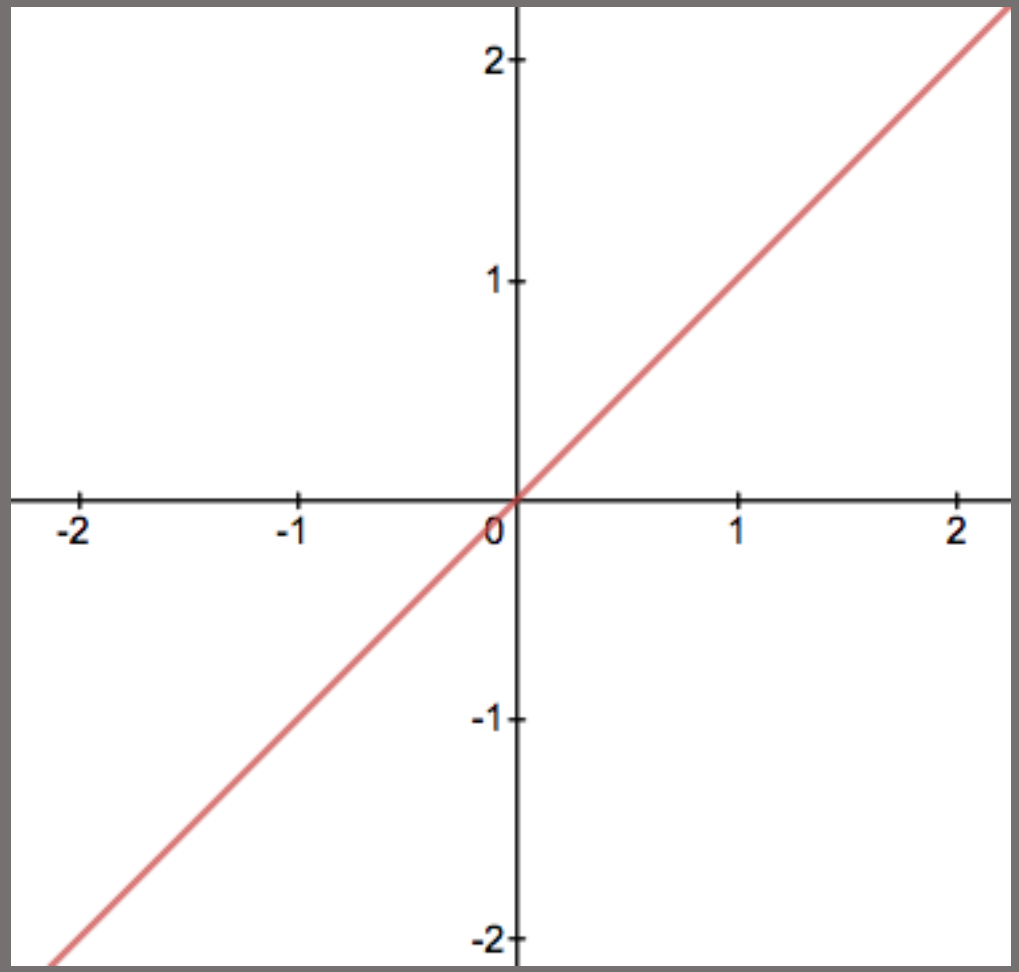


(d) $f(x) = -\sqrt{-(2x+1)}$

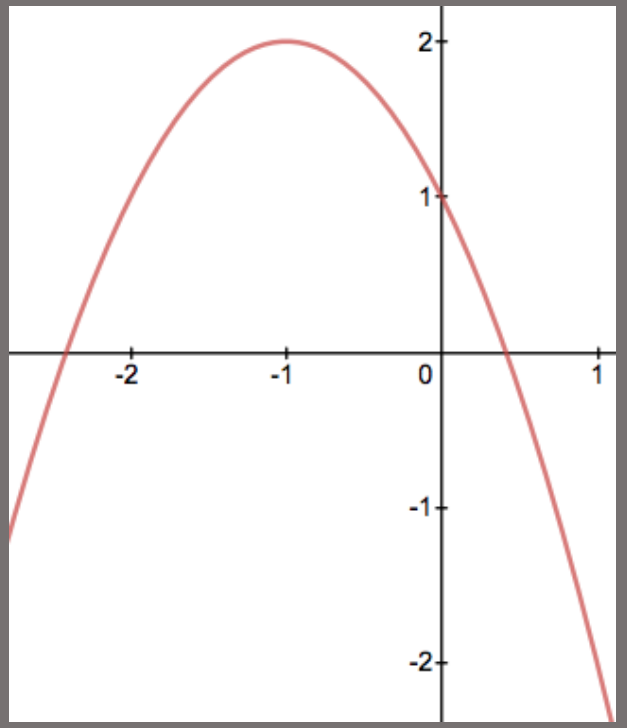


Example 3 (Polynomial Functions). Find the domain and range of the following functions.

(a) $f(x) = x$



(b) $f(x) = -x^2 - 2x + 1$



(c) $f(x) = x^3 + 5x^2 + 2x + 2$