Directed Learning Activity

Descriptive Statistical Formulas

1. Order of Operations

2. Evaluate basic statistical formula

Objective 1: Order of Operations

Many statistical formulas involve simplifying expressions containing multiple operations, parenthesis, mathematical symbols, etc. This process is known as order of operations.

Steps

1. Simplify the expression inside the parenthesis or any grouping symbols.
2. Evaluate the exponents.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Note: An acronym that is often used to remember the order of operations is PEMDAS. **P**arenthesis, **E**xponents, **M**ultiplication/**D**ivision, **A**ddition/**S**ubtraction.

Examples:

Simplify.

1. $-7 + 11^2 - 5(-4)$

Solution:

$= -7 + \mathbf{11^2} - 5(-4)$	Evaluate the exponent
$= -7 + 121 - \mathbf{5(-4)}$	Multiply
$= -7 + 121 + 20$	Add and Subtract
$= 134$	

2. $-2(7 - 4^2) - 24 \div 6(-2)$

Solution:

$= -2(7 - \mathbf{4^2}) - 24 \div 6(-2)$	Simplify the exponent inside the parenthesis
$= -2(\mathbf{7 - 16}) - 24 \div 6(-2)$	Simplify the expression inside the parenthesis

$= \mathbf{-2(-9)} - 24 \div 6(-2)$ Multiply and Divide from left to right

$= 18 - \mathbf{24 \div 6}(-2)$

$= 18 - \mathbf{4(-2)}$

$= 18 + 8$ Add

$= 26$

3. $\frac{(7-5)^2+(3-5)^2+(9-5)^2}{5-1}$

Solution:

$= \frac{(\mathbf{7-5})^2+(\mathbf{3-5})^2+(\mathbf{9-5})^2}{5-1}$ Follow the order of operations to simplify the top and bottom expressions. Evaluate each parenthesis on the top.

$= \frac{\mathbf{(2)^2+(-2)^2+(4)^2}}{5-1}$ Evaluate the exponents

$= \frac{\mathbf{4+4+16}}{\mathbf{5-1}}$ Simplify the top and bottom expressions

$= \frac{\mathbf{24}}{\mathbf{4}}$ Divide

$= 6$

Your turn:

Simplify. Show all your steps.

1. $8^2 - 36 \div 9(-2)$

2. $150-2(2\cdot 6-4)^2$

3. $161-8[6(6)-6^2]+2^2(5)$

4. $\frac{\left(4^3-2\right)+7}{5(2+4)-7}$

Objective 2: Evaluate Basic Statistical Formula

A common symbol in many statistical formulas is the sigma notation, $\boldsymbol{\Sigma}$. It represents the sum or summation symbol. When you see this symbol, it means we will be adding values. Let us learn how to evaluate expressions containing the sigma notation, $\boldsymbol{\Sigma}$.

Examples:

Evaluate. (Round to two decimal places if necessary)

1. $\sum_{i=1}^{5} i^2$

Solution:
It is the sum of $\boldsymbol{i^2}$ starting with 1, then 2, then 3, until you reach 5.

$\sum_{i=1}^{5} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

$= 1 + 4 + 9 + 16 + 25$

$= 55$

2. Let $x_i = 2, 5, 7, 13, 20$ and $n = 5$

$$\frac{\sum x_i + \sum x_i^2}{n}$$

Solution:
First, let us understand what each sigma expression means.

$\sum \boldsymbol{x_i}$ means add all the x_i values

$\sum \boldsymbol{x_i^2}$ means square each of the x_i values then add

When evaluating formulas, a common practice in Statistics is to organize the data on a table.

x_i	x_i^2
2	4
5	25
7	49
13	169
20	400
$\Sigma x_i = 47$	$\Sigma x_i^2 = 647$

$$\frac{\sum x_i + \sum x_i^2}{n} = \frac{47+647}{5}$$
$$= \frac{694}{5}$$
$$= 138.8$$

Your turn:

Evaluate. Round to two decimal places if necessary. (You may use a calculator)

1. $\sum_{k=2}^{7}(k-1)^2$

2. Let $x_i = 3, 6,7,9,11,12$ and $n = 6$

$\frac{\sum x_i^2 - \sum x_i}{n}$ (Round your answer to two decimal places if necessary)

Definition:

Population – All subjects of interest or being studied

Sample – A group of some subject selected from a population.

For example,

Let the data set, $x_i = 25, 32, 42, 45, 52$ represents the ages of millionaires in the United States. This data set is a **sample** since there are more than five millionaires in the U.S. A population would be a list of the ages of ALL the millionaires in the U.S.

Finding the Mean

A statistical data that is often used in Statistics is the mean or average. To calculate the mean, add all the values in the data set then divide by the total numbers in the data set. We use the symbol μ if the data set is from a population, and $\bar{\bar{x}}$ if the data set is from a sample.

<u>**Formulas**</u>

Population Mean: $\mu = \frac{\sum x_i}{N}$

where x_i represents the data value and N represents the total number of values in the population.

Sample Mean: $\bar{x} = \frac{\sum x_i}{n}$

where x_i represents the data value and n represents the total number of values in the sample.

Example:

The data set are the quiz scores of five students from a class of 35 students. Find the mean. (Round to two decimal places if necessary).

$x_i = 7,13,15,16,19$

Since the data set is a sample of $n = 5$ students, we will use the symbol $\bar{x}$ to calculate the mean.

$$\bar{x} = \frac{\sum x_i}{n} = \frac{7+13+15+16+19}{5} = \frac{70}{5} = 14$$

Finding the variance and standard deviation

Another statistical data that you will learn to compute in Statistics are variance and standard deviation. They are measurements of the spread of the data. The square root of the variance is the standard deviation.

Examples:

1. Find the standard deviation if the variance is 81.

Solution:
The standard deviation is $\sqrt{81} = 9$

2. Find the variance if the standard deviation is 5.4.

Solution:
If the square root of the variance is the standard deviation, then raising the standard deviation to the 2nd power gives you the variance.

Variance = $5.4^2 = 29.16$

Your turn:

3. Given the population variance $\sigma^2 = 4.3$, find the standard deviation.
(Write down the symbol used and round answer to two decimal places if necessary)

4. Given the sample standard deviation $s = 3.5$, find the variance.
(Write down the symbol used)

Let us now discuss the formulas for variance and standard deviation. The statistical formulas for calculating the variance and standard deviation are:

Population Variance:

$$\sigma^2 = \frac{\sum(x_i-\mu)^2}{N} \qquad \text{OR} \qquad \sigma^2 = \frac{N(\sum x_i^2)-(\sum x_i)^2}{N^2} \text{ (Alternative Formula)}$$

Population Standard Deviation:

$$\sigma = \sqrt{\frac{\sum(x_i-\mu)^2}{N}} \qquad \text{OR} \qquad \sigma = \sqrt{\frac{N(\sum x_i^2)-(\sum x_i)^2}{N^2}}$$

Sample Variance:

$$s^2 = \frac{\sum(x_i-\bar{x})^2}{n-1} \qquad \text{OR} \qquad s^2 = \frac{n(\sum x_i^2)-(\sum x_i)^2}{n(n-1)} \text{ (Alternative Formula)}$$

Sample Standard Deviation:

$$s = \sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}} \qquad \text{OR} \qquad s = \sqrt{\frac{n(\sum x_i^2)-(\sum x_i)^2}{n(n-1)}}$$

Example:

The weights (in lbs.) of five randomly selected pit bulls are:
26, 32, 45, 35, 42

a. Determine if the data set is a population or sample.
b. Find the mean. (Round answer to two decimal places if necessary)
c. Find the variance and standard deviation. (Round answer to two decimal places if necessary).

Solution:

a. Since there are more than five pit bulls in the world, the data set represents a sample.

b. Use the sample mean formula

$$\bar{x} = \frac{\sum x_i}{n} = \frac{26+32+45+35+42}{5} = \frac{180}{5} = 36 \text{ lbs}$$

c. Since the data set is from a sample, use the sample variance and standard deviation formulas. Two formulas are provided. We will use the alternative formula and organize the data using a table.

x_i	x_i^2
26	676
32	1024
45	2025
35	1225
42	1764
$\Sigma x_i = 180$	$\Sigma x_i^2 = 6714$

Sample Variance:

$$s^2 = \frac{n(\sum x_i^2)-(\sum x_i)^2}{n(n-1)} = \frac{5(6714)-(180)^2}{5(5-1)} = \frac{1170}{20} = 58.5 \text{ lbs}^2$$

Sample Standard Deviation:

$$s = \sqrt{\frac{n(\sum x_i^2) - (\sum x_i)^2}{n(n-1)}} = \sqrt{\frac{5(6714) - (180)^2}{5(5-1)}} = \sqrt{\frac{1170}{20}} = \sqrt{58.5} = 7.64852\ldots$$

= 7.65 lbs(Rounded to two decimal places)

Your turn:

5. The ages (in years) of all family Smith's children are:
17, 14, 16, 10, 8

a. Determine if the data set is a population or sample.
b. Find the mean. (Write down the formula used and round your answer to two decimal places if necessary)
c. Find the variance and standard deviation. (Write down the formula used and round your answer to two decimal places if necessary).

6. Seven students are randomly selected from a Statistic class. Their heights are measured in inches: 73, 65, 68, 64, 75, 67, 62

a. Determine if the data set is a population or sample.
b. Find the mean. (Write down the formula used and round your answer to two decimal places if necessary)
c. Find the variance and standard deviation. (Write down the formula used and round your answer to two decimal places if necessary).